

$$f(x) = x^2 + 2x + 1 \quad \& \quad g(x) = 2x - 3$$

① $f \circ g$

$$\begin{aligned} (f \circ g)(x) &= f(g(x)) = f(2x - 3) \\ &= (2x - 3)^2 + 2(2x - 3) + 1 \\ &= 4x^2 - 12x + 9 + 4x - 6 + 1 \\ &= 4x^2 - 8x + 4 \end{aligned}$$

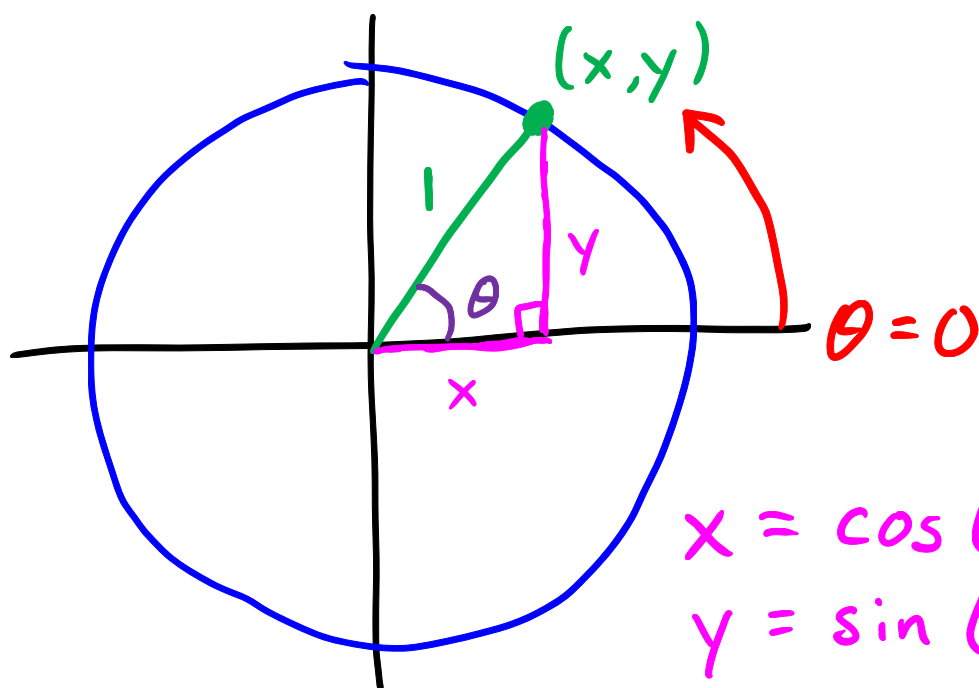
② $g \circ f$

$$\begin{aligned} (g \circ f)(x) &= g(x^2 + 2x + 1) = 2(x^2 + 2x + 1) - 3 \\ &= 2x^2 + 4x - 1 \end{aligned}$$

③ $g \circ g \circ g$

$$\begin{aligned} (g \circ g \circ g)(x) &= g(g(g(x))) = g(g(2x - 3)) \\ &= g(\underbrace{2(2x - 3) - 3}_{(g \circ g)(x)}) = g(4x - 9) \\ &= 2(4x - 9) - 3 = 6x - 21 \end{aligned}$$

$$\cos(e^{x^2}) = f(g(h(x)))$$



measure
 θ in
radians
($2\pi = 360^\circ$)

$$x = \cos \theta$$

$$y = \sin \theta$$

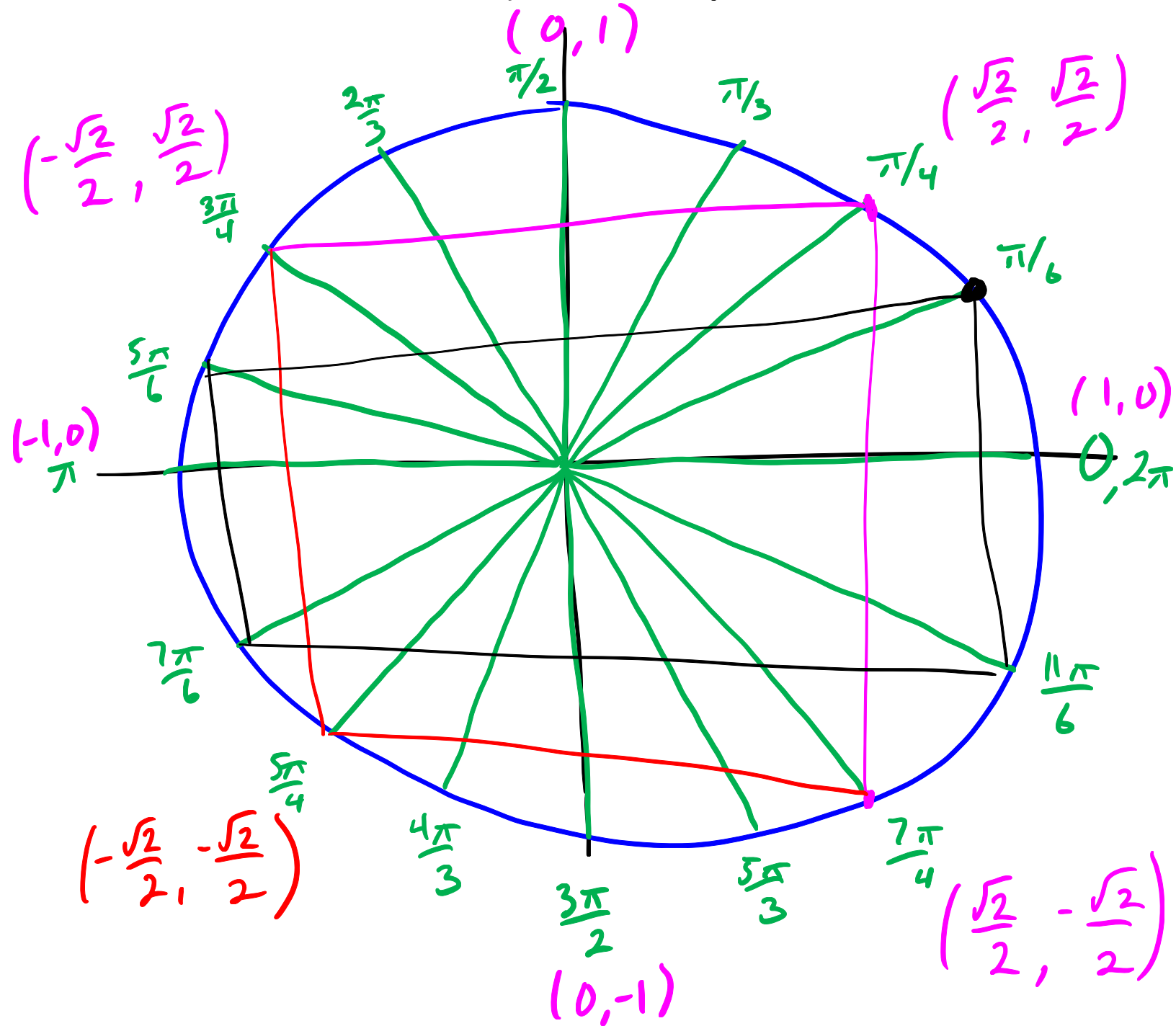
θ	0°	30°	45°	60°	90°
	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin \theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos \theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0

θ	0°	30°	45°	60°	90°
	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin \theta$	$\sqrt{0}/2$	$\sqrt{1}/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	$\sqrt{4}/2$
$\cos \theta$	$\sqrt{4}/2$	$\sqrt{3}/2$	$\sqrt{2}/2$	$\sqrt{1}/2$	$\sqrt{0}/2$

simplify

$$\tan \theta = \frac{\sin \theta}{\cos \theta}, \quad \sec \theta = \frac{1}{\cos \theta}, \quad \csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$$

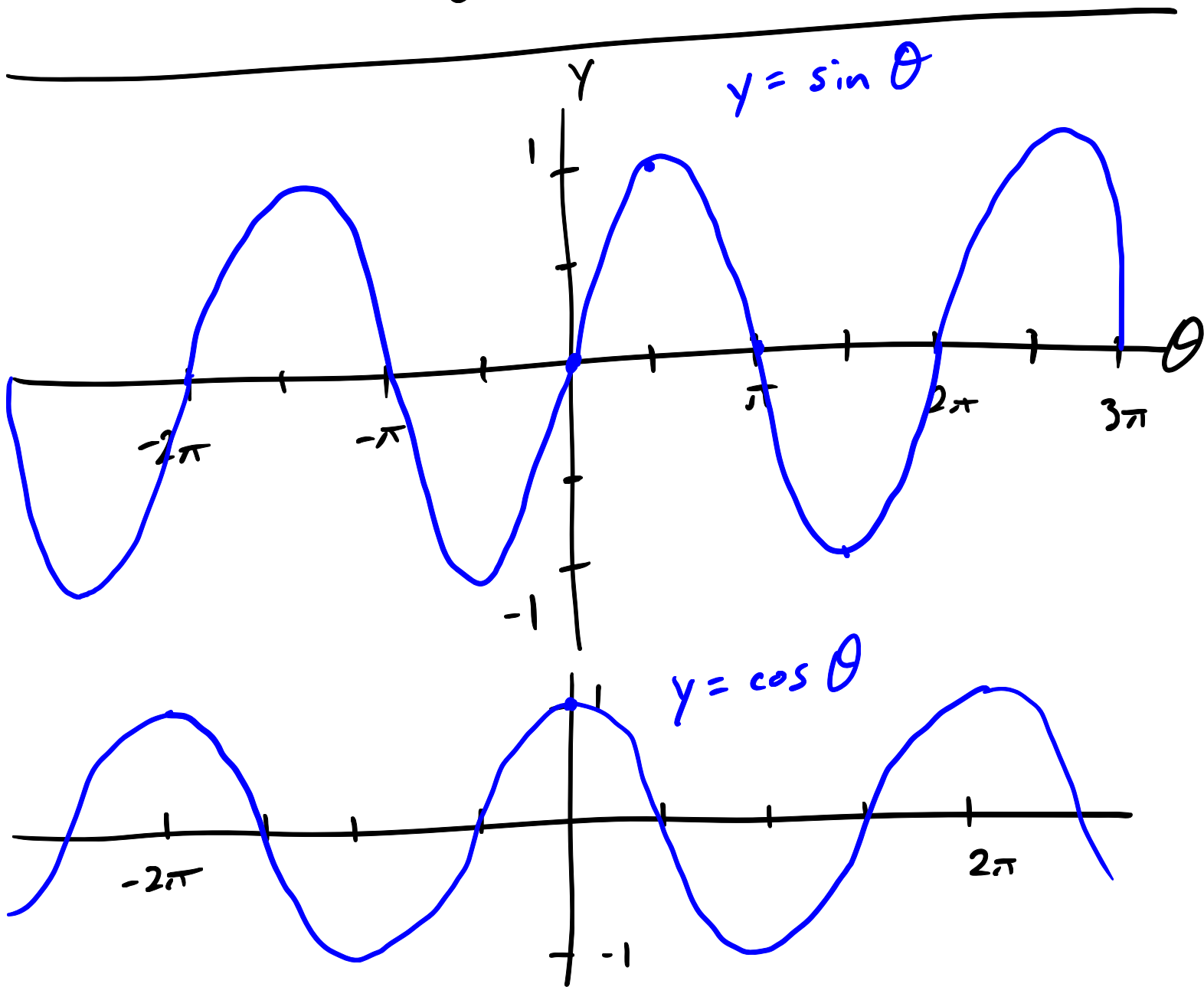


$$x^2 + y^2 = 1$$

$$\underline{\text{Ex:}} \left. \begin{aligned} \sin \frac{\pi}{6} &= \frac{1}{2} \\ \cos \frac{\pi}{6} &= \frac{\sqrt{3}}{2} \end{aligned} \right\} \rightarrow \begin{aligned} \sin \frac{5\pi}{6} &= \frac{1}{2} \\ \cos \frac{5\pi}{6} &= -\frac{\sqrt{3}}{2} \end{aligned}$$

$$\swarrow \quad \searrow$$

$$\frac{7\pi}{6} \qquad \frac{11\pi}{6}$$



$$\sin(\theta + 2\pi) = \sin \theta$$

$$\cos(\theta + 2\pi) = \cos \theta$$

$$\begin{cases} \sin(\theta + 2n\pi) = \sin \theta \\ \cos(\theta + 2n\pi) = \cos \theta \end{cases}$$

$$n = \dots, -2, -1, 0, 1, 2, \dots$$

$$\sin\left(\frac{17\pi}{3}\right) = \sin\left(\frac{11\pi}{3} + \frac{6\pi}{3}\right) = \sin\left(\frac{5\pi}{3} + \cancel{\frac{12\pi}{3}}^{4\pi}\right)$$

$$2\pi = \frac{6\pi}{3}$$

$$= \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

$$f(x) = \underline{x^2 + 2x + 1} \quad g(x) = \underline{2x - 3}$$

⑥ $f \circ g$ composition

$$\begin{aligned} \underline{(f \circ g)(x)} &= \underline{f(g(x))} = f(2x-3) \\ &= (2x-3)^2 + 2(2x-3) + 1 \\ &= 4x^2 - 12x + 9 + 4x - 6 + 1 \\ &= \boxed{4x^2 - 8x + 4} \end{aligned}$$

⑦ $g \circ f$

$$(g \circ f)(x) = g(f(x)) = 2(x^2 + 2x + 1) - 3 = 2x^2 + 4x - 1$$

⑧ $g \circ g \circ g$

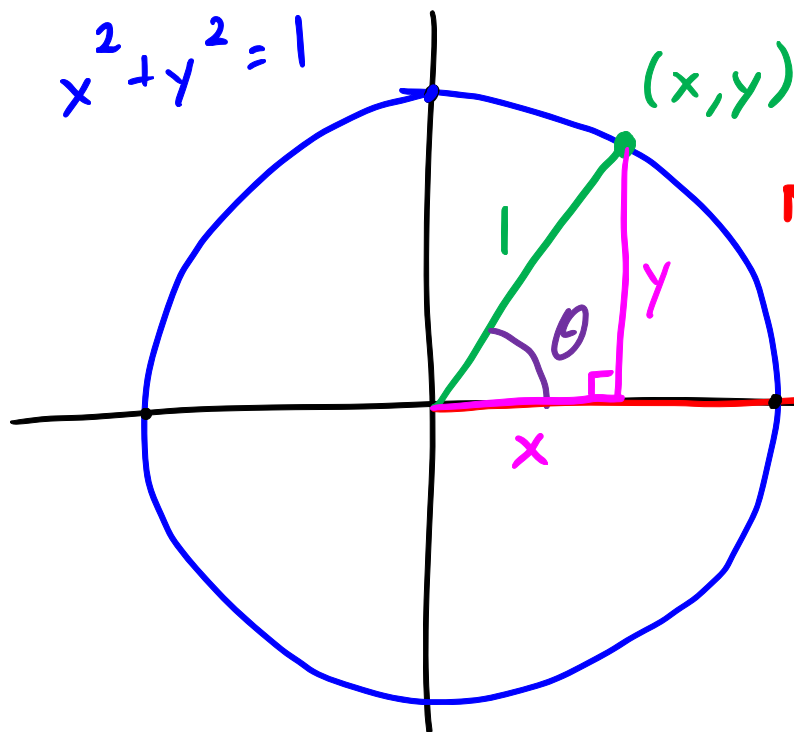
$$\begin{aligned} (g \circ g \circ g)(x) &= g(g(\underline{g(x)})) \\ &= g(g(\underline{2x-3})) \\ &= g(\underline{2(2x-3)-3}) = g(4x-9) \\ &= 2(4x-9) - 3 = 8x - 21 \end{aligned}$$

$$\cos(e^{x^2})$$

$$f(g(h(x)))$$

$$\begin{array}{ccc} \nearrow & \nearrow & \uparrow \\ \cos x & e^x & x^2 \end{array}$$

$$x^2 + y^2 = 1$$



$$x = \cos \theta$$

$$y = \sin \theta$$

$$\theta = 0$$

$$2\pi = 360^\circ$$

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin \theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos \theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0

simplifying

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin \theta$	$\sqrt{0}/2$	$\sqrt{1}/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	$\sqrt{4}/2$
$\cos \theta$	$\sqrt{4}/2$	$\sqrt{3}/2$	$\sqrt{2}/2$	$\sqrt{1}/2$	$\sqrt{0}/2$

Other trig functions

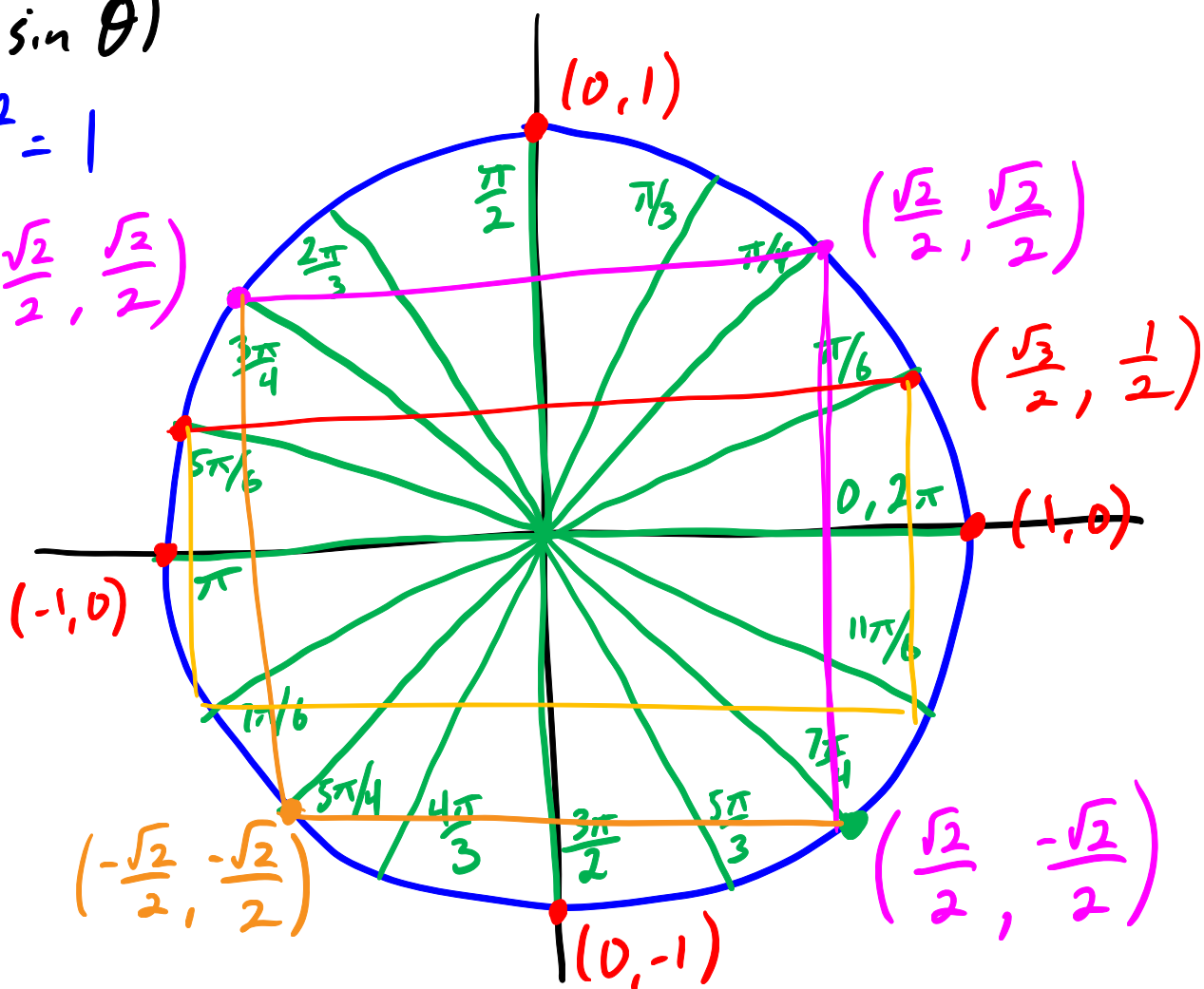
$$\tan \theta = \frac{\sin \theta}{\cos \theta}, \quad \sec \theta = \frac{1}{\cos \theta}, \quad \csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$$

$$(\cos \theta, \sin \theta)$$

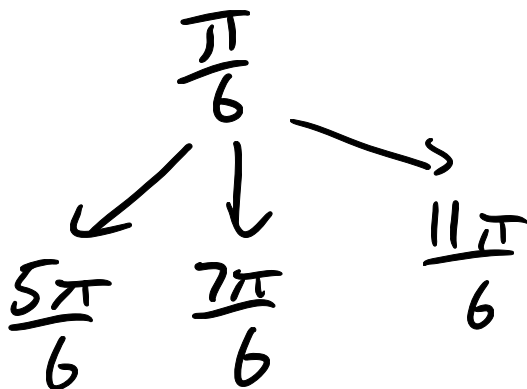
$$x^2 + y^2 = 1$$

$$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

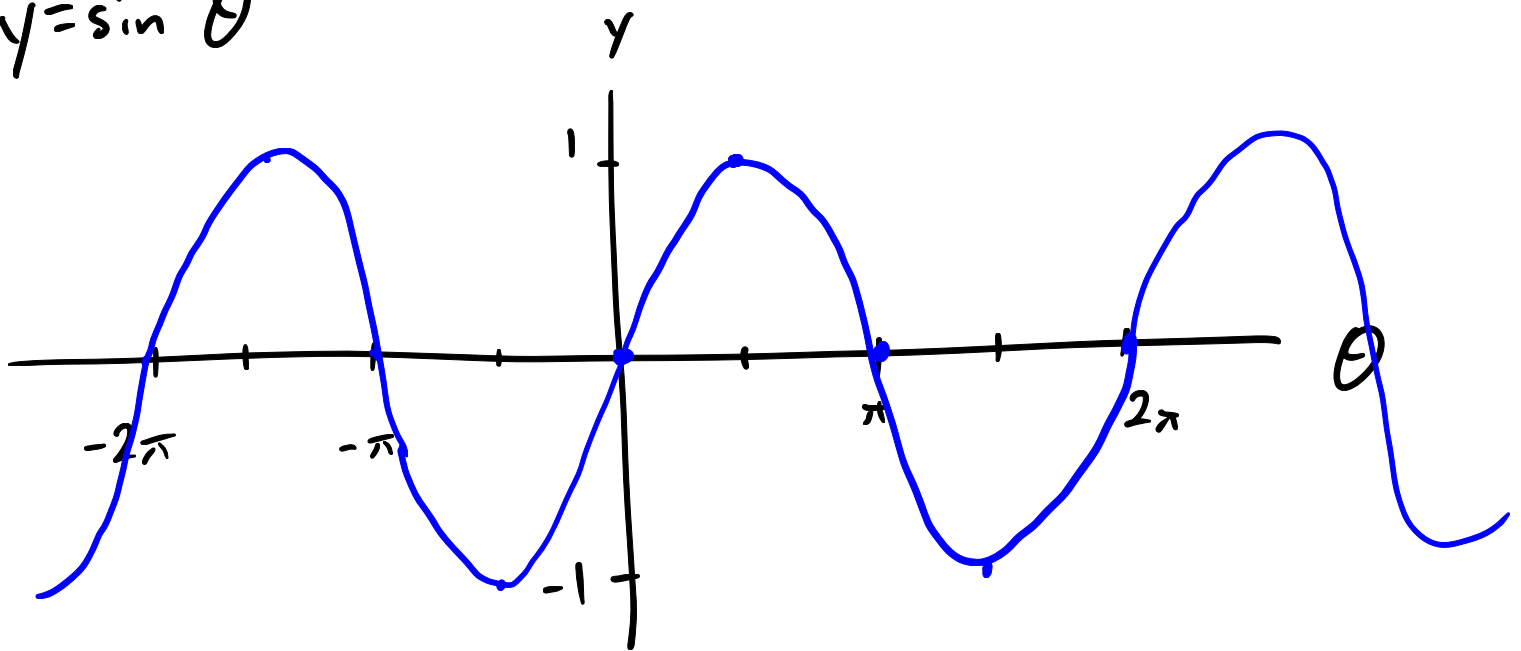


Ex: $\cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$

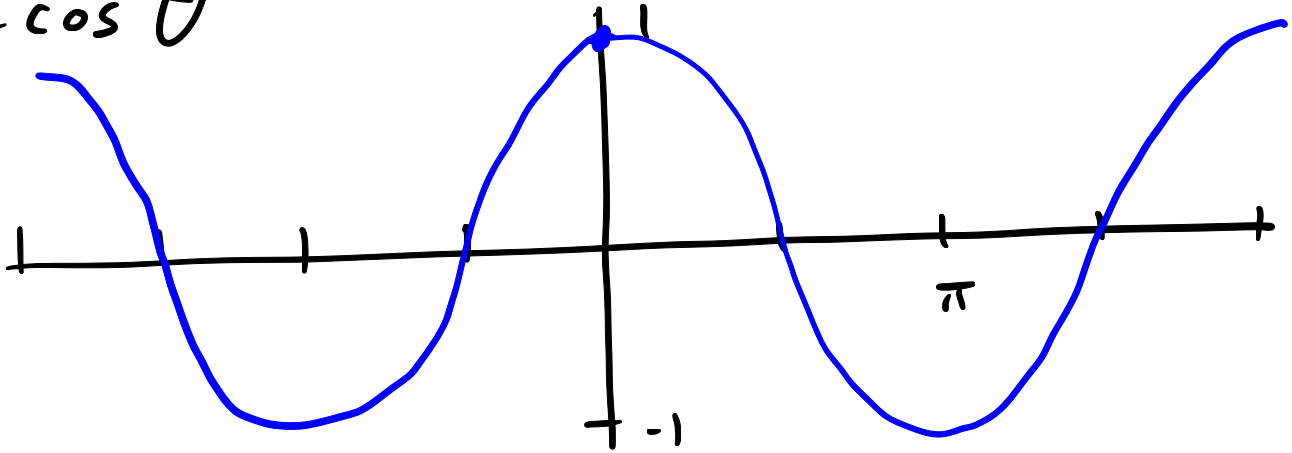
$$\sin \frac{5\pi}{6} = \frac{1}{2}$$



$$y = \sin \theta$$



$$y = \cos \theta$$



$$\cos\left(\frac{11\pi}{3}\right) = \cos\left(\frac{5\pi}{3} + \frac{6\pi}{3}\right)$$

$$= \cos\left(\frac{5\pi}{3} + \cancel{2\pi}\right)$$

$$= \cos\left(\frac{5\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\sin(\theta + 2n\pi) = \sin(\theta)$$

$$\cos(\theta + 2n\pi) = \cos(\theta)$$